KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

Robotics I, WS 2024/2025

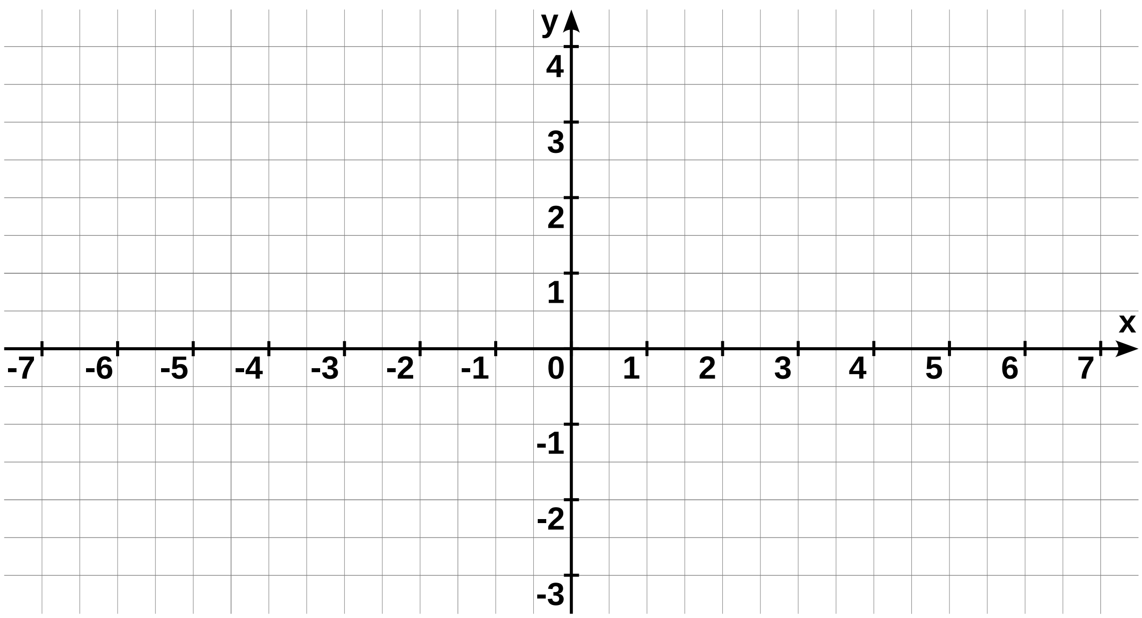
Exercise Sheet 2

Prof. Dr.-Ing. Tamim Asfour
Adenauerring 12, Geb. 50.19
Web: https://www.humanoids.kit.edu/

Date: November 13, 2024

Exercise 1 (Transformations)

In this task, the root pose of a robot and its relation to the basis coordinate system (BCS) is considered. In a second step, the relation to a target pose is included.



1. **Interpretation of a Pose**

   Let the initial root pose of the robot, described in the BCS, be given as

$$^{BCS}T_{root} = \begin{pmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & 0 & 3 \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   i.) Which transformation does $^{BCS}T_{root}$ describe?

   ii.) At which position is the robot's root located? Draw the position in the BCS.

   To describe the orientation of the robot, we use the orthonormal basis vectors of the local coordinate system tied to the robot's root. We define the x-axis as pointing into the direction of motion, the y-axis to point left, and the z-axis to point upwards.

   iii.) Write down the orthonormal basis vectors $\mathbf{v}_{x,root}$, $\mathbf{v}_{y,root}$ and $\mathbf{v}_{z,root}$ and the vector of the origin $\mathbf{v}_{o,root}$.

   iv.) Calculate the representation of these vectors in BCS. Use the result to draw the robot's root coordinate system in the BCS.

2. **Conversion Between Coordinate Systems**

   A target pose $^{BCS}T_{target}$ is given as

$$^{BCS}T_{target} = \begin{pmatrix} 0 & -1 & 0 & 3+\sqrt{3} \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   i.) Draw the target pose in the BCS.

   ii.) Calculate the pose of the target in the robot's root coordinate system.

   iii.) Calculate the pose of the robot root in the target coordinate system.

3. **Local and Global Transformation**

   Whether a transformation is applied form the left or from the right is important.

   i.) Write down a transformation T that corresponds to a 60° rotation around the z-axis.

   ii.) Apply T to $^{BCS}T_{root}$ from the **left**. Draw the resulting coordinate system $^{BCS}T_{trans,L}$.

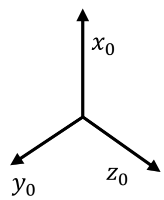
   iii.) Apply T to $^{BCS}T_{root}$ from the **right**. Draw the resulting coordinate system $^{BCS}T_{trans,R}$.

   iv.) Which of both transformations corresponds to a rotation around local axes, and which one around global axes?

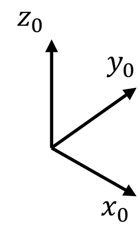
Exercise 2 (Denavit Hartenberg)

1. Transform the given right-handed coordinate systems into new coordinate systems using the provided DH parameters. Sketch the solution, all auxiliary coordinate systems as well as all parameters that lead to the solution.

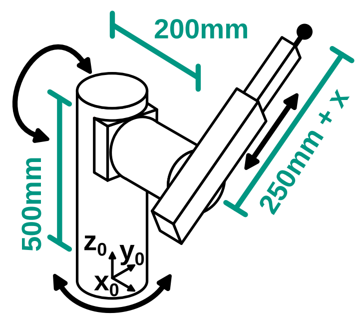
   i.) Given are the coordinate system $(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$ and the DH parameters $\theta_1 = 0°$, $d_1 = 60\,\text{mm}$, $a_1 = 0\,\text{mm}$, $\alpha_1 = 180°$:

   ii.) Given are the coordinate system $(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$ and the DH parameters $\theta_1 = 90°$, $d_1 = -30\,\text{mm}$, $a_1 = 60\,\text{mm}$, $\alpha_1 = -90°$:

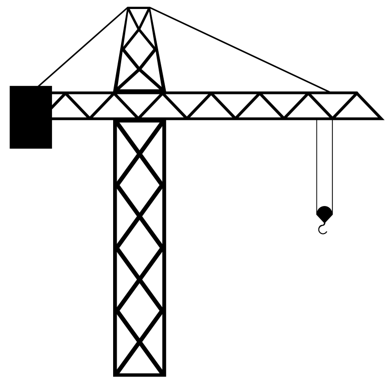
2. The following schematic representation of a robot arm is given. The basis coordinate system BCS is defined by $(\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$. Determine the DH parameters of the joints J1–J3, using the basis coordinate system BCS, and fill in the following table:



Joint	$\boldsymbol{\theta}$ [°]	$\boldsymbol{d}$ [mm]	$\boldsymbol{a}$ [mm]	$\boldsymbol{\alpha}$ [°]
J1				
J2				
J3				

Exercise 3 (Jacobian Matrix)

Consider the following crane[1] with a rotary degree of freedom at its basis and two translational degrees of freedom at the crane boom:



The crane can rotate by 360° and has a height of 20 m between floor and crane boom. The crane boom is 15 m long. The trolley stops 2 m away from the rotation axis. The hook can be lowered until reaching the ground.

1. Determine the DH parameters of the crane and the resulting transformation matrix of the end effector.

2. Determine the Jacobian matrix of the end effector.

3. The configurations $\mathbf{q}_1$ and $\mathbf{q}_2$ are given, as well as the joint velocities $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$.

$$\mathbf{q}_1 = \begin{pmatrix} 90 \\ 10 \\ 10 \end{pmatrix}, \quad \mathbf{q}_2 = \begin{pmatrix} 180 \\ 2 \\ 15 \end{pmatrix}, \quad \mathbf{p}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{p}_2 = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{p}_3 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

Determine the end effector velocities resulting from the following combinations of the crane configuration and the joint velocities.

i.) $\mathbf{q}_1, \mathbf{p}_1$

ii.) $\mathbf{q}_1, \mathbf{p}_2$

iii.) $\mathbf{q}_2, \mathbf{p}_2$

iv.) $\mathbf{q}_2, \mathbf{p}_3$

[1] Image source: https://thenounproject.com/term/crane/2225/, mirrored (CreativeCommons Attribution 3.0)